

# Two-particle correlations and the small-x gluon four- point function

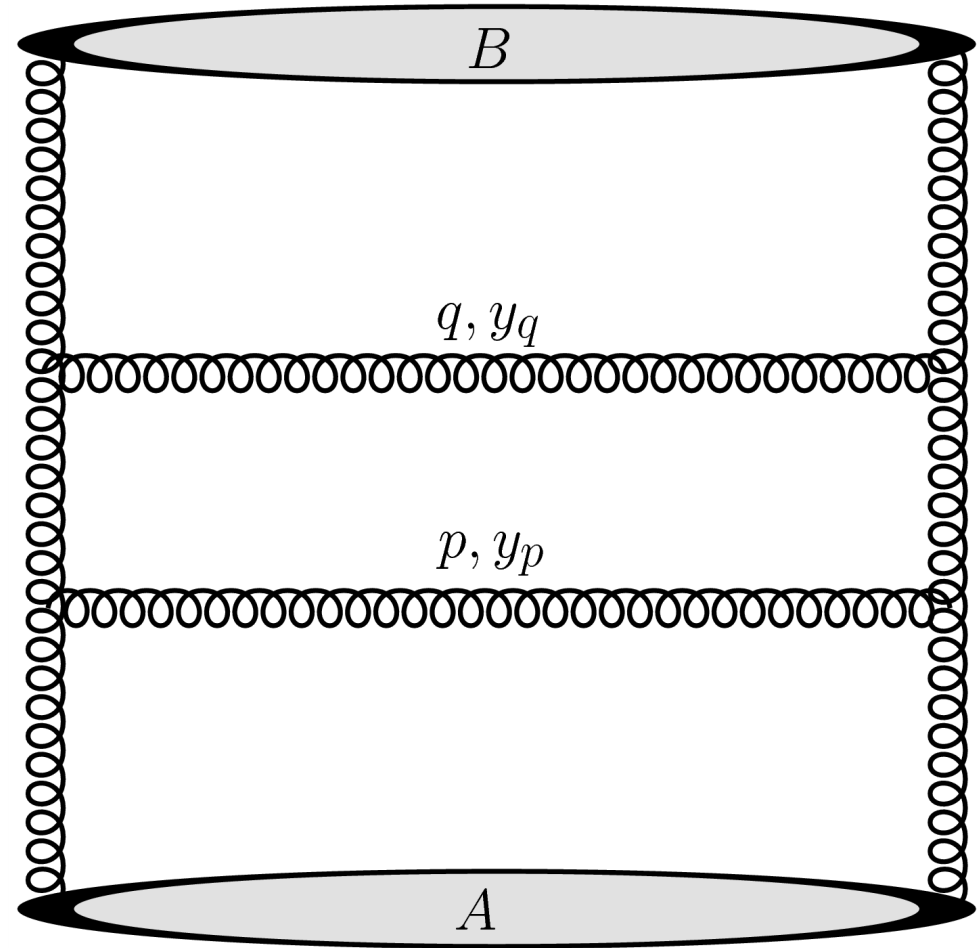
Adrian Dumitru  
RIKEN-BNL and Baruch College

RBRC Workshop  
Progress in High- $p_T$  Physics at RHIC  
March 17<sup>th</sup> - 19<sup>th</sup> 2010

[A.D., Jamal Jalilian-Marian: arXiv:1001.4820](#)

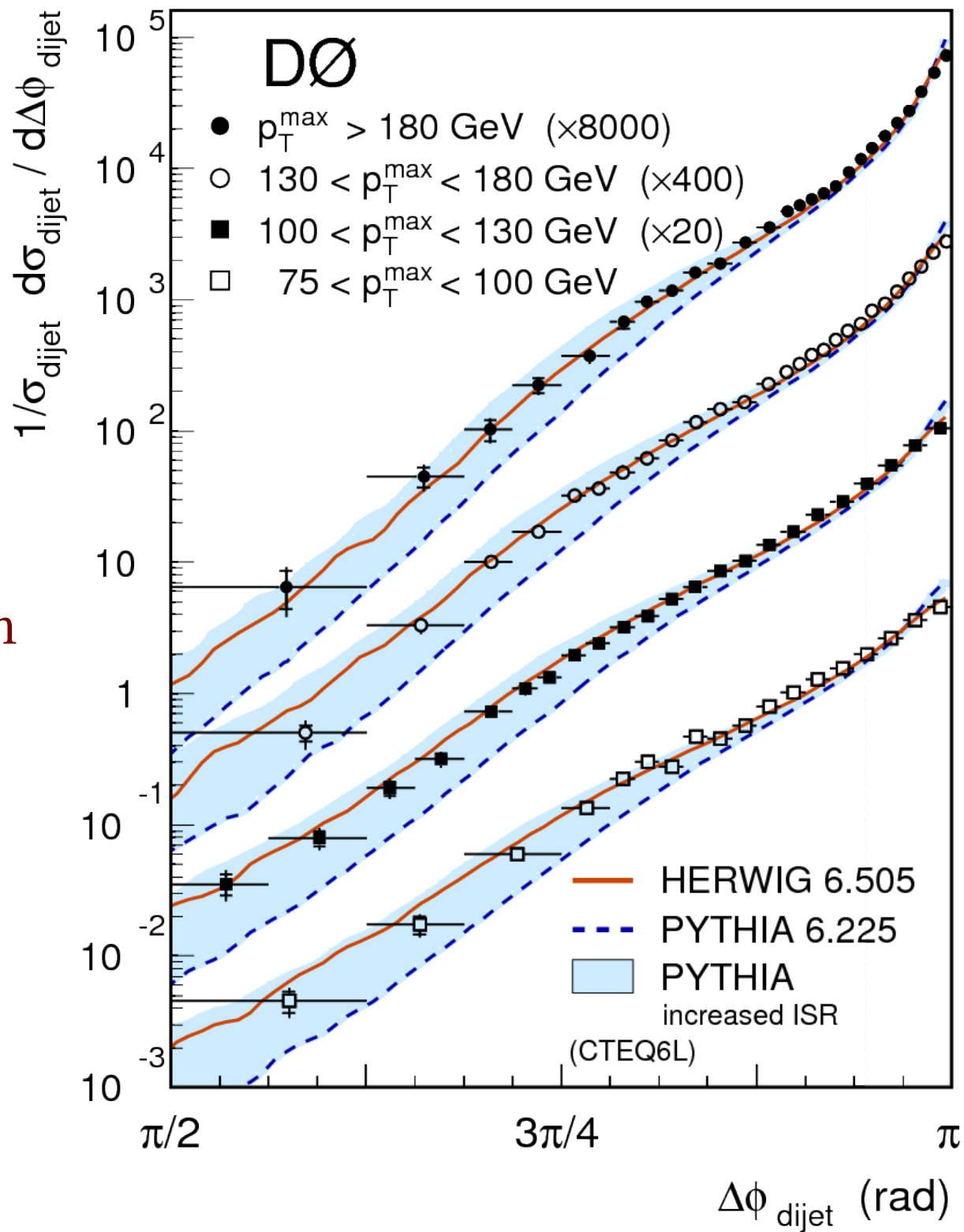
# (Correlated) two-particle production: the DGLAP way

$\sim \delta(p+q)$  (at leading order,  
back-to-back dijet)



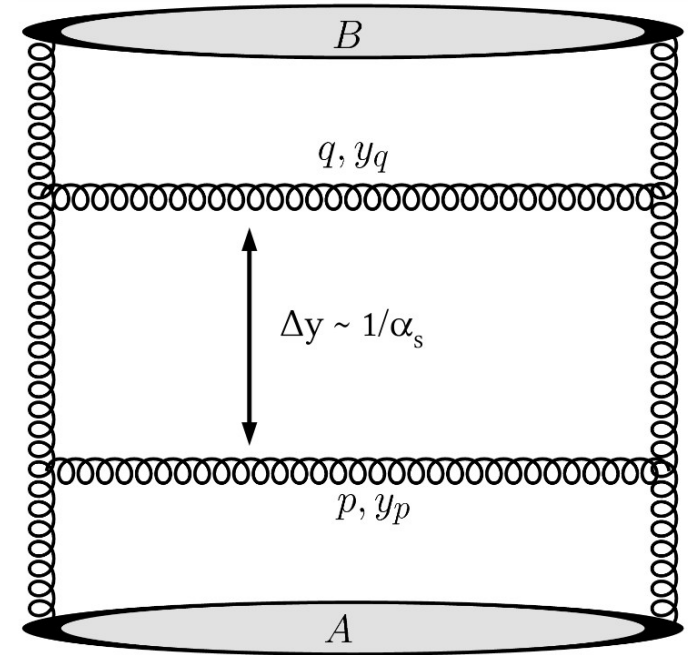
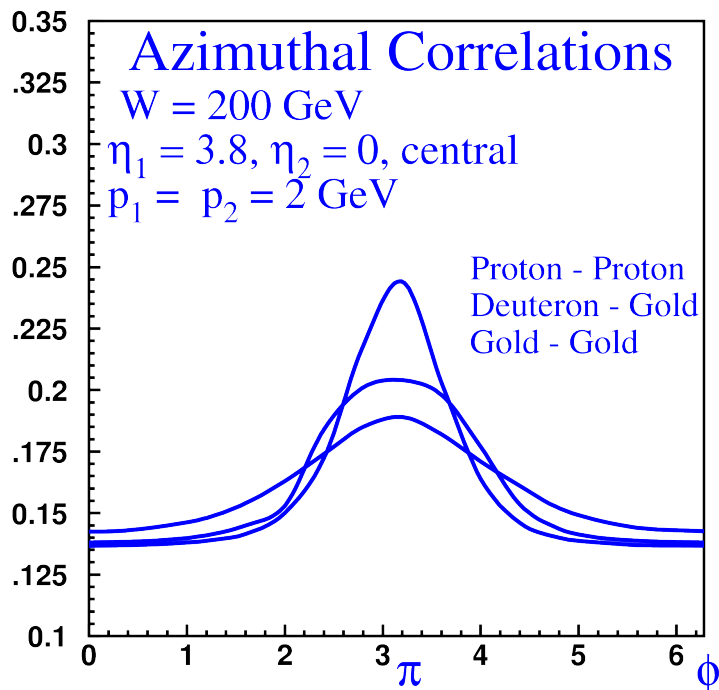
Angular correlation at high  $p_T$  ( $p\bar{p}$  @ TEVATRON) :

- DGLAP<sub>LO</sub>-ish  $\sim \delta(p+q)$



# (Correlated) two-particle production: BFKL kinematics, Mueller-Navelet jets

- smears out the back-to-back dijet

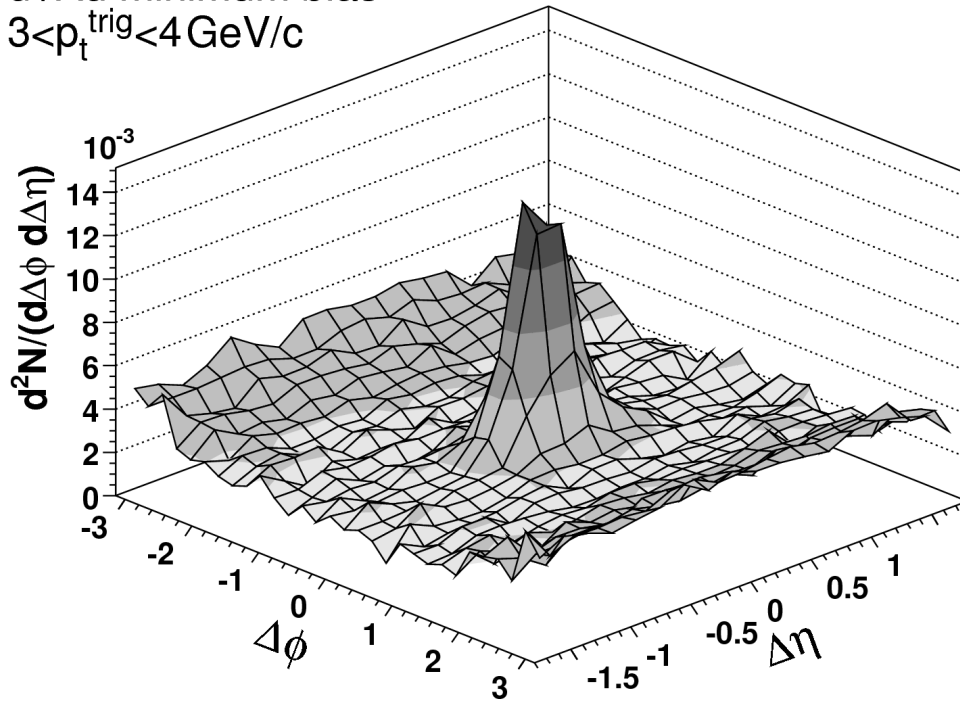


Kharzeev, Levin, McLerran:  
[hep-ph/0403271](https://arxiv.org/abs/hep-ph/0403271)

However, this talk is NOT about  
“back to back” correlations ...

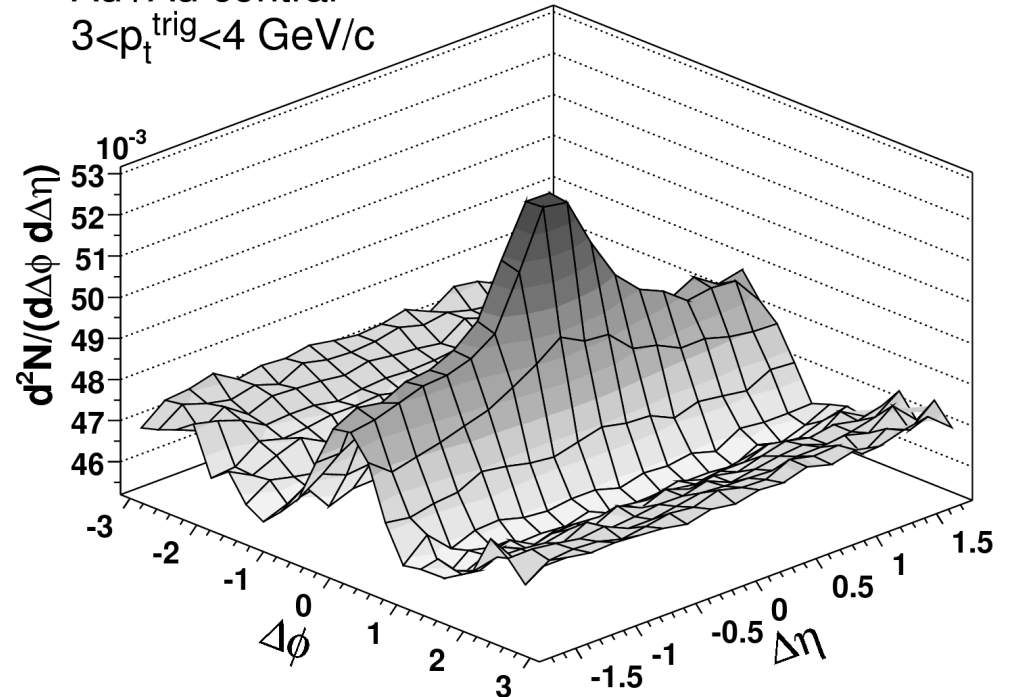
# Near-side correlations, $\Phi < 1$ (the “ridge”)

d+Au minimum bias  
 $3 < p_t^{\text{trig}} < 4 \text{ GeV}/c$

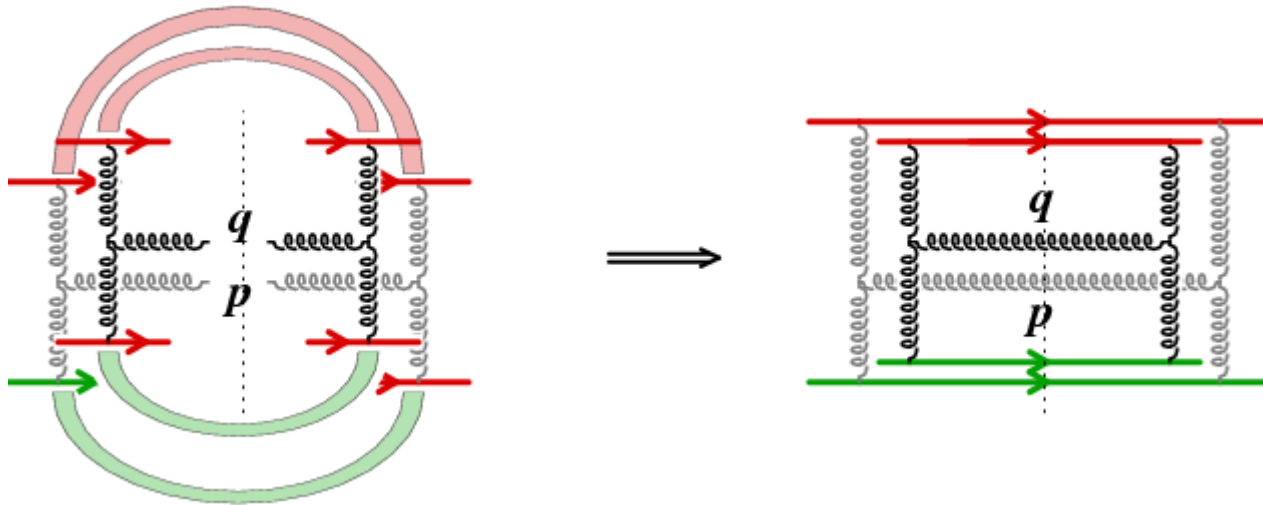


STAR (arXiv:0909.0191)

Au+Au central  
 $3 < p_t^{\text{trig}} < 4 \text{ GeV}/c$

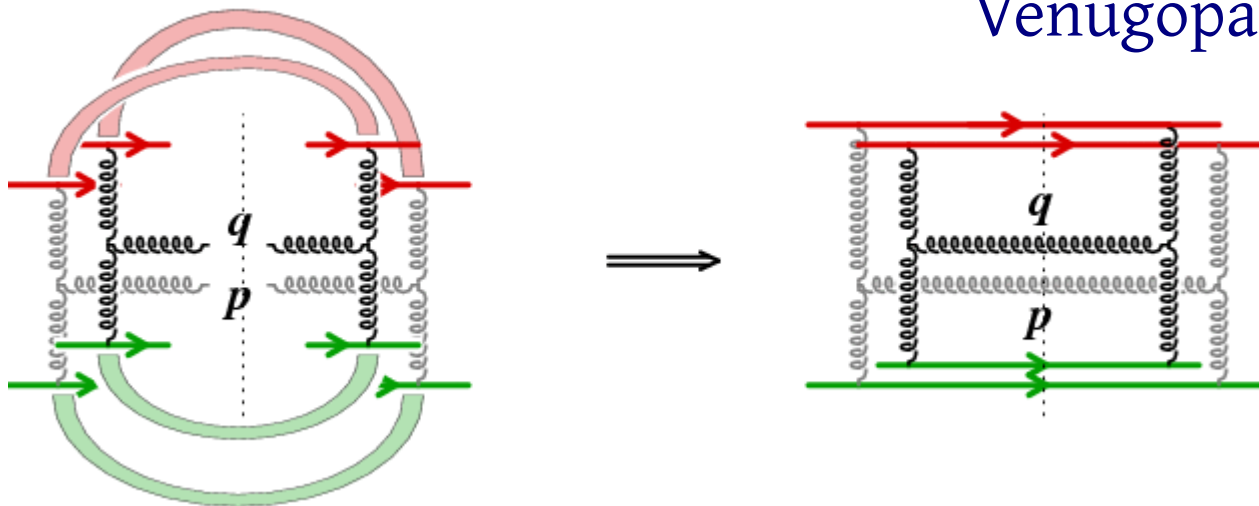


Independent production of two gluons:

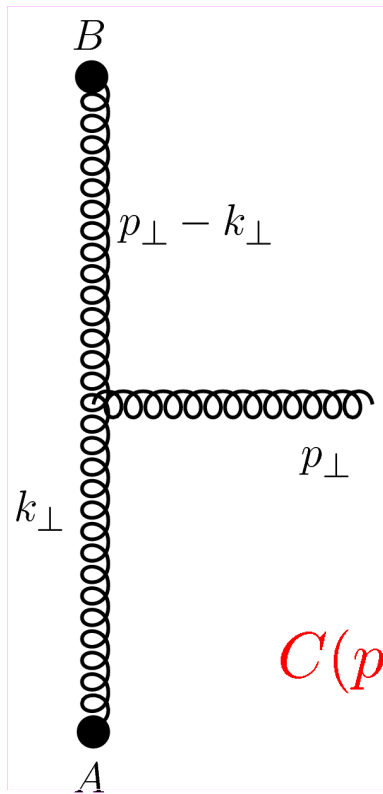


PYTHIA: “independent  
multi-parton inter-  
actions”

Correlated two-gluon production:



A.D., Gelis, McLerran,  
Venugopalan: 0804.3858



$$\mathcal{M}_\lambda^a = p^2 A^{\mu,a}(p) \epsilon_\mu^\lambda(p)$$

$$p^2 A^{\mu,a}(p) = -i f^{abc} \frac{g^3}{2} \int \frac{d^2 k_\perp}{(2\pi)^2} L^\mu(p_\perp, k_\perp) \frac{\rho_A^a(k_\perp)}{k_\perp^2} \frac{\rho_B^b(p_\perp - k_\perp)}{(p_\perp - k_\perp)^2}$$

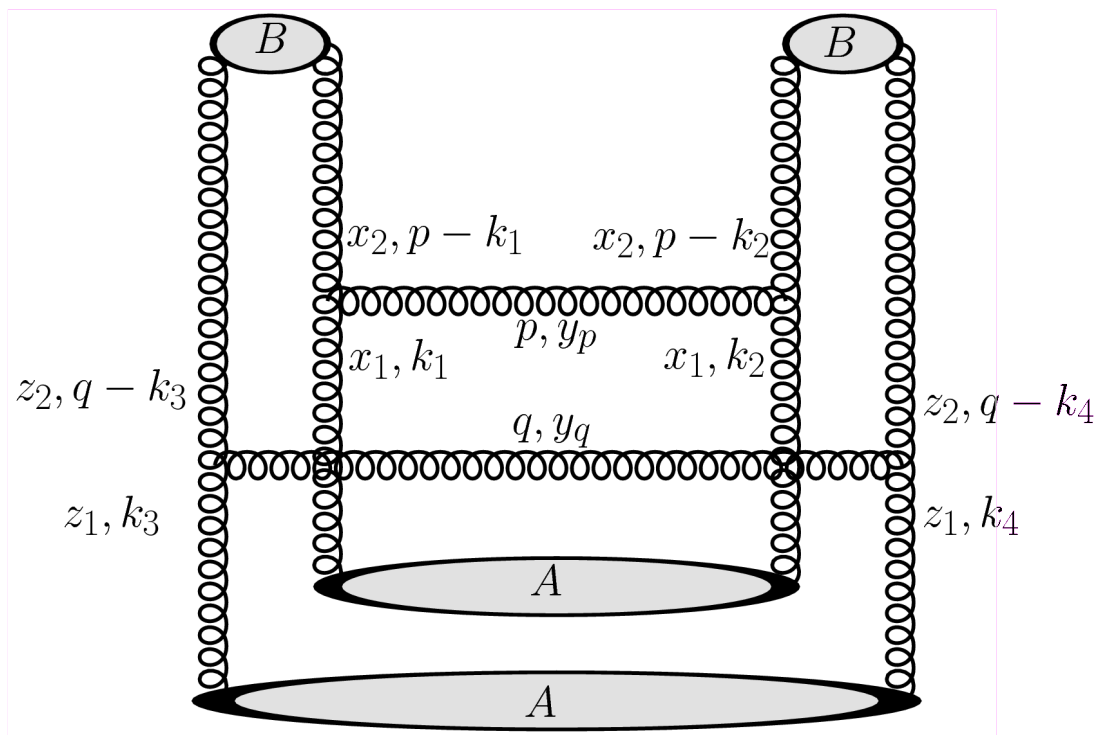
$$C(p, q) \sim \sum_{a,a';\lambda,\lambda'} \left( \left\langle |\mathcal{M}_{\lambda\lambda'}^{aa'}(p, q)|^2 \right\rangle - \left\langle |\mathcal{M}_\lambda^a(p)|^2 \right\rangle \left\langle |\mathcal{M}_{\lambda'}^{a'}(q)|^2 \right\rangle \right)$$

two-point function:

$$\langle \rho^{*a}(k) \rho^b(q) \rangle(x) \sim \frac{1}{g^2} \frac{\delta^{ab}}{N_c^2 - 1} \delta(k - q) \Phi(x, k^2) \leftarrow$$

unintegr. gluon distrib. —

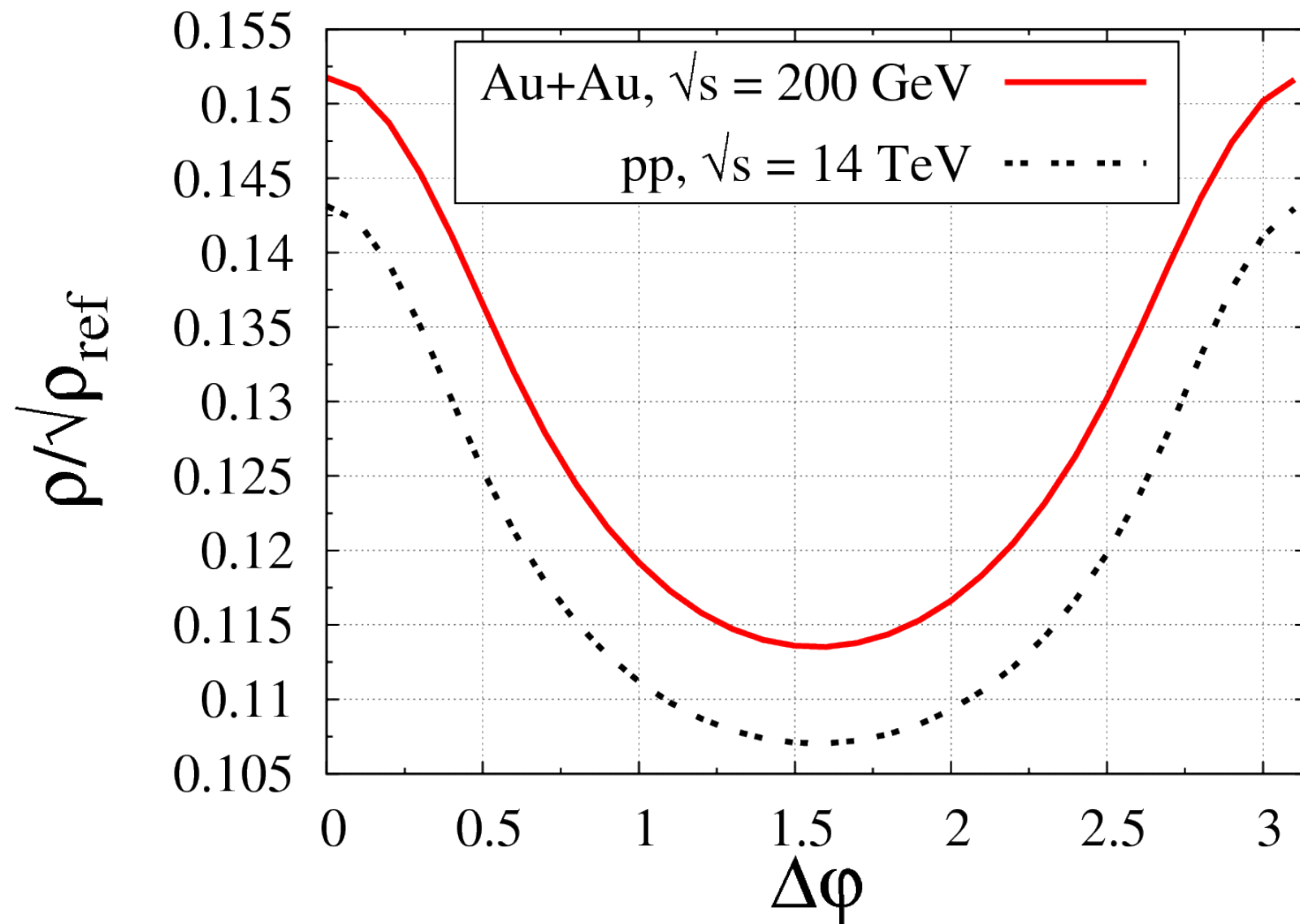




$$C(p, q) = 16(2\pi)^2 \alpha_s^2 S_\perp \frac{N^2}{(N^2 - 1)^3} \frac{1}{p_\perp^2} \frac{1}{q_\perp^2} \int d^2 k_\perp \frac{\Phi_A(x_1, (p_\perp + k_\perp)^2)}{(p_\perp + k_\perp)^2} \frac{\Phi_A(x_1, (q_\perp - k_\perp)^2)}{(q_\perp - k_\perp)^2} \frac{\Phi_B^2(x_2, k_\perp^2)}{k_\perp^4}$$

**Depends on angle  $\angle(p_\perp, q_\perp)$ , not flat in  $\phi$  !**

RHIC(Au+Au) versus LHC(pp) :

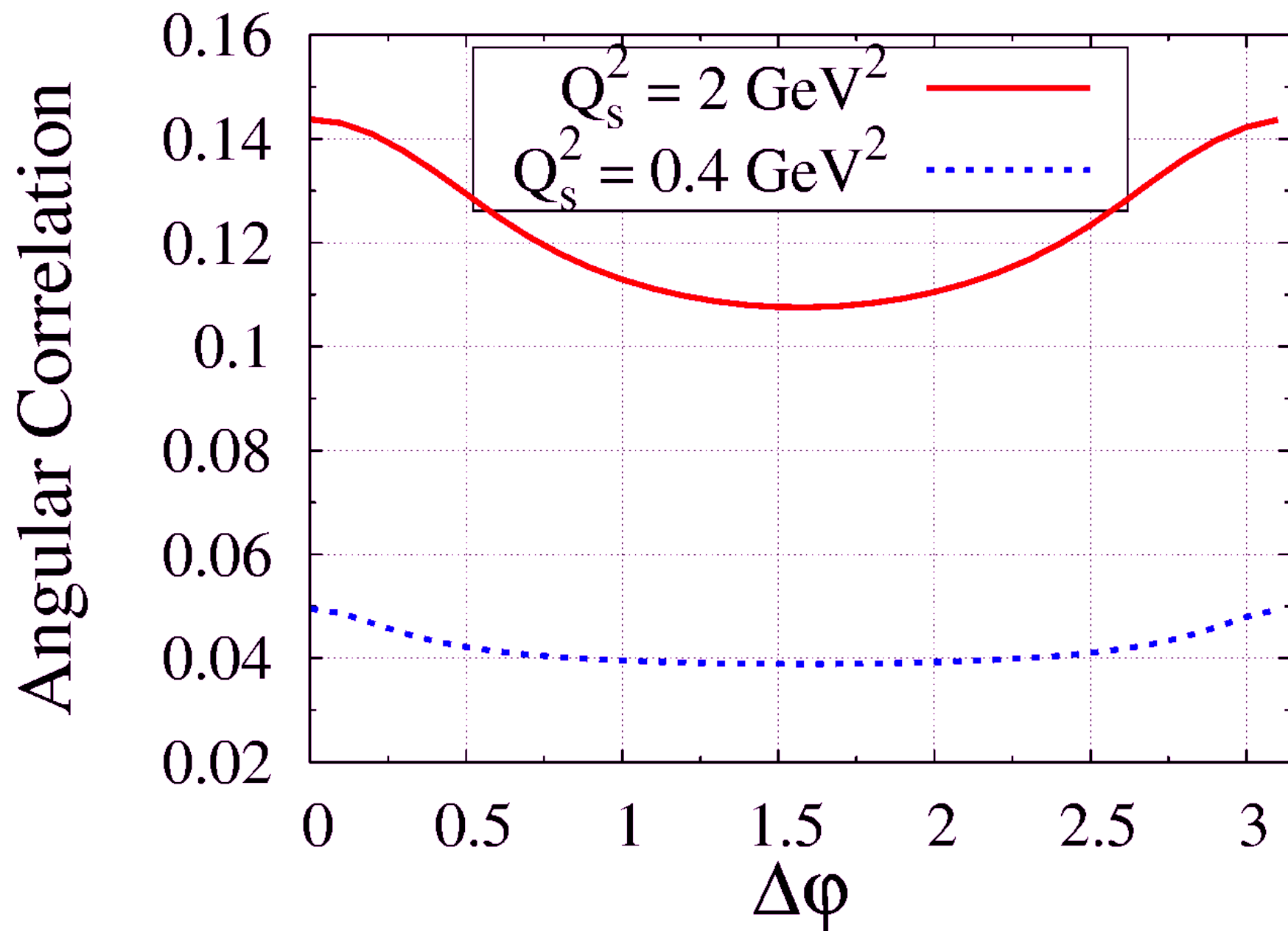


ridge in pp @ LHC ?!

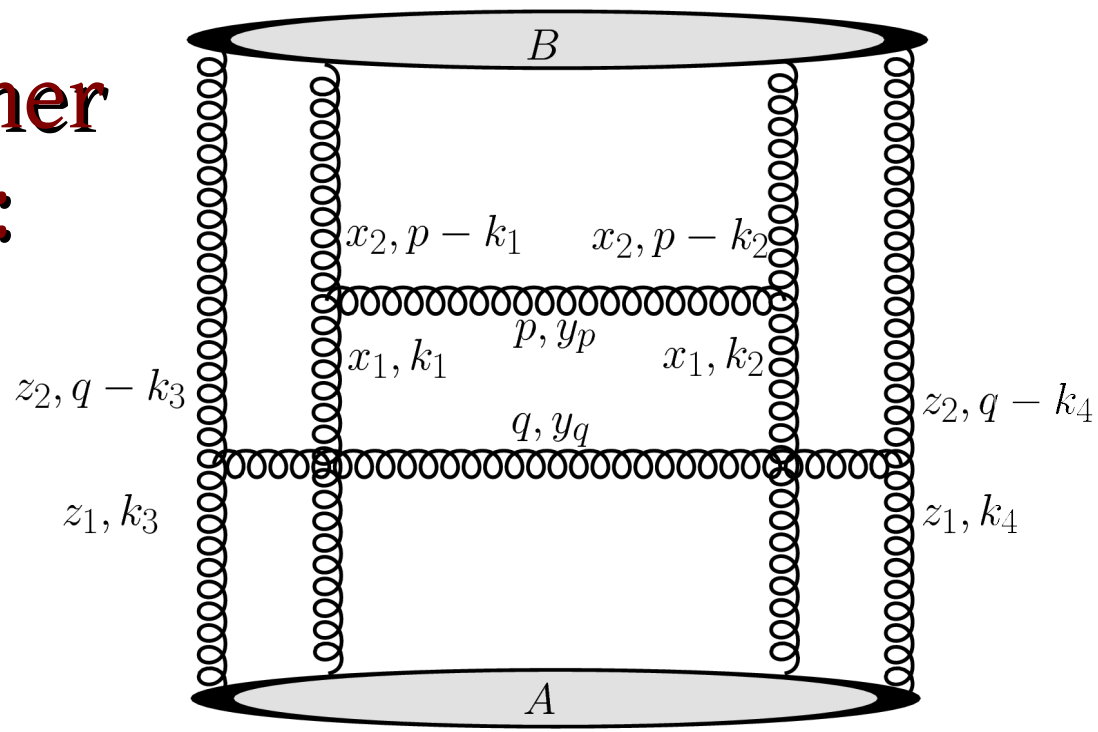
AA versus pp :

$$p_{\perp} = 5 \text{ GeV}, \quad y_p = y_q = 0$$

$$q_{\perp} = 4 \text{ GeV}$$



however, we should rather  
compute THIS diagram:



$$\begin{aligned}
 C(p_{\perp}, q_{\perp}) = & \frac{g^{12}}{64(2\pi)^6} (f_{abc} f_{a'\bar{b}\bar{c}} f_{a\hat{b}\hat{c}} f_{a'\tilde{b}\tilde{c}}) \int \prod_{i=1}^4 \frac{d^2 k_{i\perp}}{(2\pi)^2 k_{i\perp}^2} \\
 & \times \frac{L_{\mu}(p_{\perp}, k_{1\perp}) L^{\mu}(p_{\perp}, k_{2\perp})}{(p_{\perp} - k_{1\perp})^2 (p_{\perp} - k_{2\perp})^2} \frac{L_{\nu}(q_{\perp}, k_{3\perp}) L^{\nu}(q_{\perp}, k_{4\perp})}{(q_{\perp} - k_{3\perp})^2 (q_{\perp} - k_{4\perp})^2} \\
 & \times \left\langle \rho_1^{*\hat{b}}(k_{2\perp}) \rho_1^{*\tilde{b}}(k_{4\perp}) \rho_1^b(k_{1\perp}) \rho_1^{\bar{b}}(k_{3\perp}) \right\rangle \\
 & \times \left\langle \rho_2^{*\hat{c}}(p_{\perp} - k_{2\perp}) \rho_2^{*\tilde{c}}(q_{\perp} - k_{4\perp}) \rho_2^c(p_{\perp} - k_{1\perp}) \rho_2^{\bar{c}}(q_{\perp} - k_{3\perp}) \right\rangle
 \end{aligned}$$

# Gaussian approximation (factorization of $\langle \rho^4 \rangle \sim \langle \rho^2 \rangle^2$ )

$$\begin{aligned} \langle \rho^a \rho^b \rho^c \rho^d \rangle &= \delta^{ab} \delta^{cd} \langle \rho^2 \rangle^2 + \delta^{ac} \delta^{bd} \langle \rho^2 \rangle^2 + \delta^{ad} \delta^{bc} \langle \rho^2 \rangle^2 \\ &\quad + \mathcal{O}\left(\frac{1}{N_c}\right) \end{aligned}$$

$\langle \rho^2 \rangle$  can be obtained from BFKL or BK eqn.  
(standard unintegrated gluon distrib.)

$$\begin{aligned} \partial_Y \langle \rho^a \rho^b \rho^c \rho^d \rangle &= \delta^{ab} \delta^{cd} \mathcal{Z} + \delta^{ac} \delta^{bd} \mathcal{Z} + \delta^{ad} \delta^{bc} \mathcal{Z} \\ \mathcal{Z} &= \langle \rho^2 \rangle \mathcal{K} \otimes \langle \rho^2 \rangle \end{aligned}$$



BFKL kernel

# Complete B-JIMWLK four-point function: (no Gaussian approx.)

$$\begin{aligned}
\frac{d}{dY} \langle \alpha_r^a \alpha_{\bar{r}}^b \alpha_s^c \alpha_{\bar{s}}^d \rangle = & \\
& \frac{g^2 N_c}{(2\pi)^3} \int d^2 z \left\langle \frac{\alpha_z^a \alpha_{\bar{r}}^b \alpha_s^c \alpha_{\bar{s}}^d}{(r-z)^2} + \frac{\alpha_r^a \alpha_z^b \alpha_s^c \alpha_{\bar{s}}^d}{(\bar{r}-z)^2} + \frac{\alpha_r^a \alpha_{\bar{r}}^b \alpha_z^c \alpha_{\bar{s}}^d}{(s-z)^2} + \frac{\alpha_r^a \alpha_{\bar{r}}^b \alpha_s^c \alpha_z^d}{(\bar{s}-z)^2} - 4 \frac{\alpha_r^a \alpha_{\bar{r}}^b \alpha_s^c \alpha_{\bar{s}}^d}{z^2} \right\rangle \\
& + \frac{g^2}{\pi} \int \frac{d^2 z}{(2\pi)^2} \left\langle f^{\epsilon\kappa a} f^{f\kappa b} \frac{(r-z) \cdot (\bar{r}-z)}{(r-z)^2 (\bar{r}-z)^2} \left[ \alpha_r^e \alpha_{\bar{r}}^f - \alpha_r^e \alpha_z^f - \alpha_z^e \alpha_{\bar{r}}^f + \alpha_z^e \alpha_z^f \right] \alpha_s^c \alpha_{\bar{s}}^d \right. \\
& \quad + f^{\epsilon\kappa a} f^{f\kappa c} \frac{(r-z) \cdot (s-z)}{(r-z)^2 (s-z)^2} \left[ \alpha_r^e \alpha_s^f - \alpha_r^e \alpha_z^f - \alpha_z^e \alpha_s^f + \alpha_z^e \alpha_z^f \right] \alpha_{\bar{r}}^b \alpha_{\bar{s}}^d \\
& \quad + f^{\epsilon\kappa a} f^{f\kappa d} \frac{(r-z) \cdot (\bar{s}-z)}{(r-z)^2 (\bar{s}-z)^2} \left[ \alpha_r^e \alpha_{\bar{s}}^f - \alpha_r^e \alpha_z^f - \alpha_z^e \alpha_{\bar{s}}^f + \alpha_z^e \alpha_z^f \right] \alpha_{\bar{r}}^b \alpha_s^c \\
& \quad + f^{\epsilon\kappa b} f^{f\kappa c} \frac{(\bar{r}-z) \cdot (s-z)}{(\bar{r}-z)^2 (s-z)^2} \left[ \alpha_{\bar{r}}^e \alpha_s^f - \alpha_{\bar{r}}^e \alpha_z^f - \alpha_z^e \alpha_s^f + \alpha_z^e \alpha_z^f \right] \alpha_r^a \alpha_{\bar{s}}^d \\
& \quad + f^{\epsilon\kappa b} f^{f\kappa d} \frac{(\bar{r}-z) \cdot (\bar{s}-z)}{(\bar{r}-z)^2 (\bar{s}-z)^2} \left[ \alpha_{\bar{r}}^e \alpha_{\bar{s}}^f - \alpha_{\bar{r}}^e \alpha_z^f - \alpha_z^e \alpha_{\bar{s}}^f + \alpha_z^e \alpha_z^f \right] \alpha_r^a \alpha_s^c \\
& \quad \left. + f^{\epsilon\kappa c} f^{f\kappa d} \frac{(s-z) \cdot (\bar{s}-z)}{(s-z)^2 (\bar{s}-z)^2} \left[ \alpha_s^e \alpha_{\bar{s}}^f - \alpha_s^e \alpha_z^f - \alpha_z^e \alpha_{\bar{s}}^f + \alpha_z^e \alpha_z^f \right] \alpha_r^a \alpha_{\bar{r}}^b \right\rangle .
\end{aligned}$$

$$A^\mu(x^+, r) \equiv \delta^{\mu-} \alpha(x^+, r) = -g \delta^{\mu-} \delta(x^+) \frac{1}{\nabla_\perp^2} \rho(x^+, r) \quad k^2 \alpha(k) = g \rho(k)$$

however, “subleading- $N_c$ ” piece  
contributes at the same order to  $C(p,q)$

Complete Balitsky/JIMWLK four-point function:  
(in Gaussian approximation)

$$\langle \rho^a \rho^b \rho^c \rho^d \rangle = \delta^{ab} \delta^{cd} \langle \rho^2 \rangle^2 + \frac{1}{N_c} f^{ab\kappa} f^{cd\kappa} \langle \rho^2 \rangle^2 + \dots$$

$$f_{gaa'} f_{g'bb'} f_{gcc'} f_{g'dd'} \delta^{ac} \delta^{bd} \delta^{a'b'} \delta^{c'd'} = N_c^2 (N_c^2 - 1)$$

$$f_{gaa'} f_{g'bb'} f_{gcc'} f_{g'dd'} \frac{1}{N_c} f^{ab\kappa} f^{cd\kappa} \delta^{a'c'} \delta^{b'd'} = N_c^2 (N_c^2 - 1)$$

Projectile Target

[Note: independent/uncorrel. production

$$] \quad f_{gaa'} f_{g'bb'} f_{gcc'} f_{g'dd'} \delta^{ac} \delta^{bd} \delta^{a'c'} \delta^{b'd'} = N_c^2 (N_c^2 - 1)^2$$

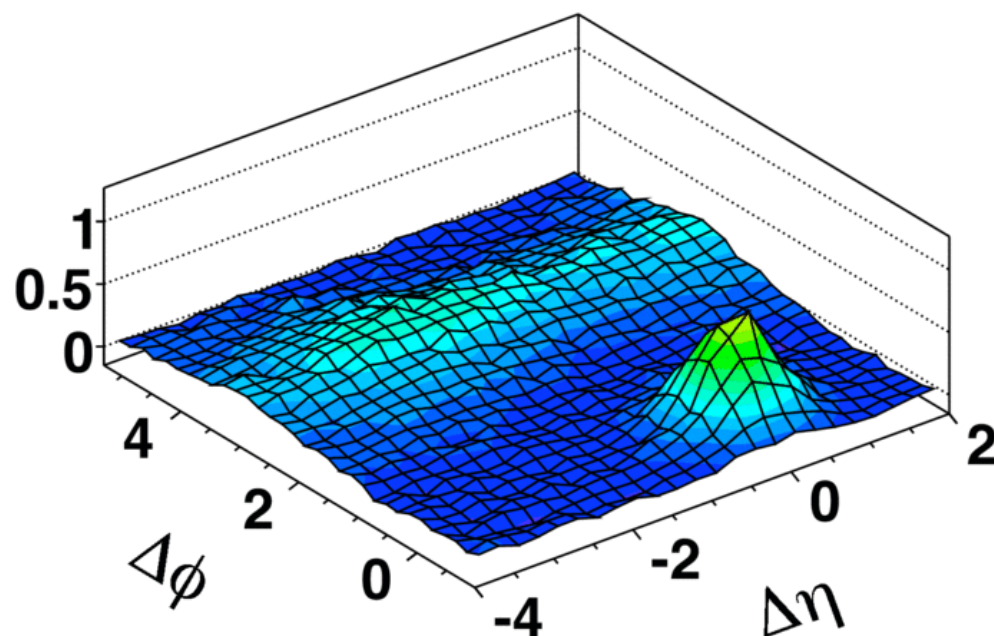
# Summary:

- kinematic regime:  
 $p, q \sim Q_s$  (say, 1-3 GeV for pp @ LHC, AA @ RHIC)
- effect disappears for small  $Q_s$  / large  $p, q$
- $\Phi \ll \pi$
- particle correlations probe complete B-JIMWLK evolution equation incl. “ $N_c$  corrections”  
(unlike single-inclusive cross-section !)
- should be interesting for pp @ LHC



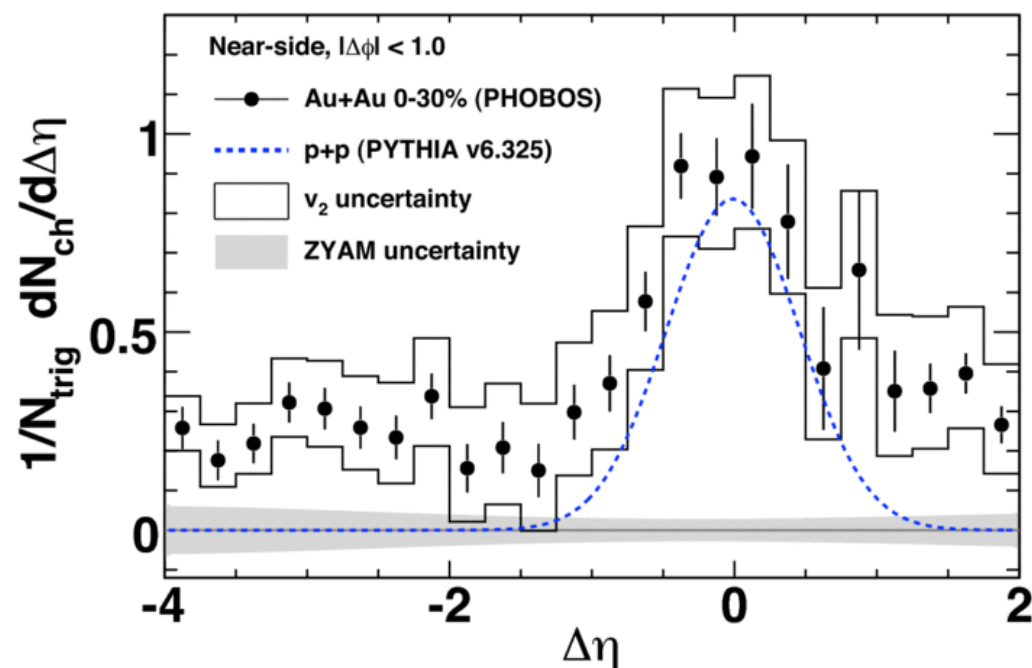
BACKUP SLIDES

PHOBOS (arXiv:0903.2811):



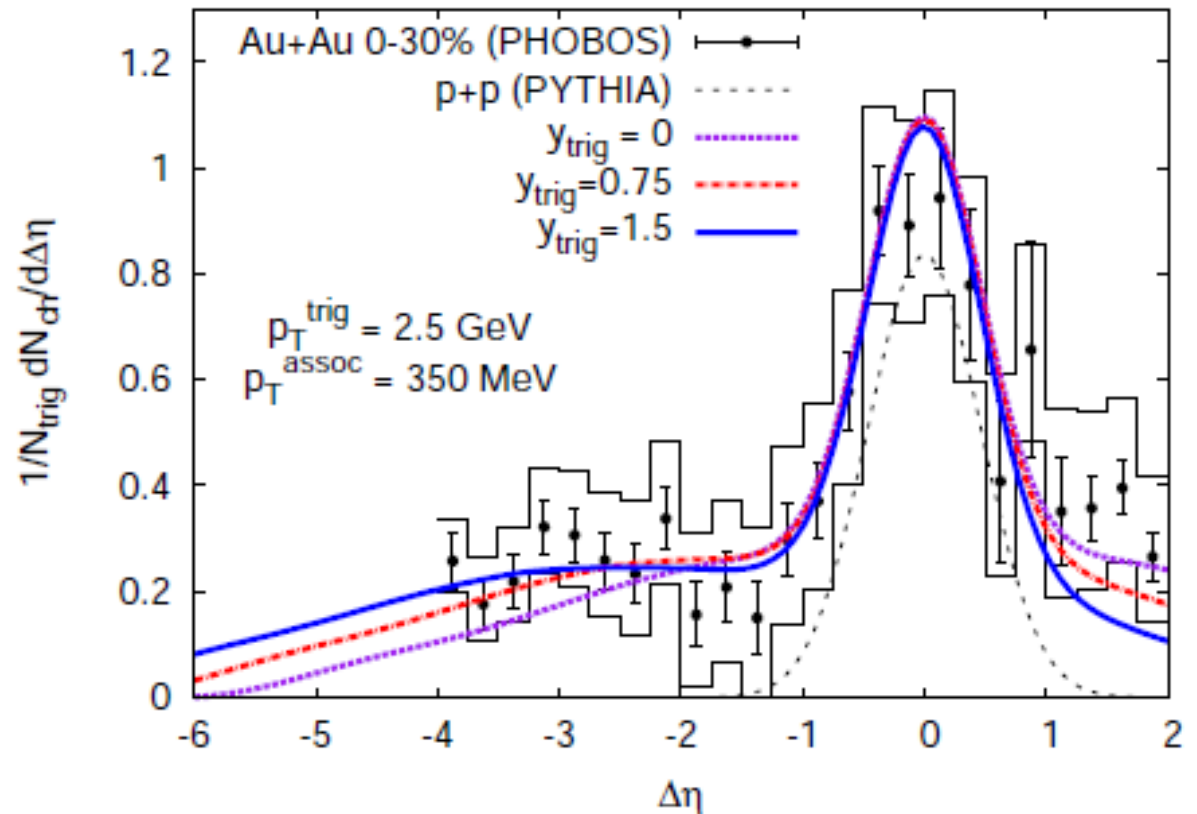
PYTHIA pp,  $p_T^{\text{trig}} > 2.5 \text{ GeV}$

PHOBOS Au+Au, 0-30%,  
 $p_T^{\text{trig}} > 2.5 \text{ GeV}$



# long-range rapidity evolution and small-x evolution

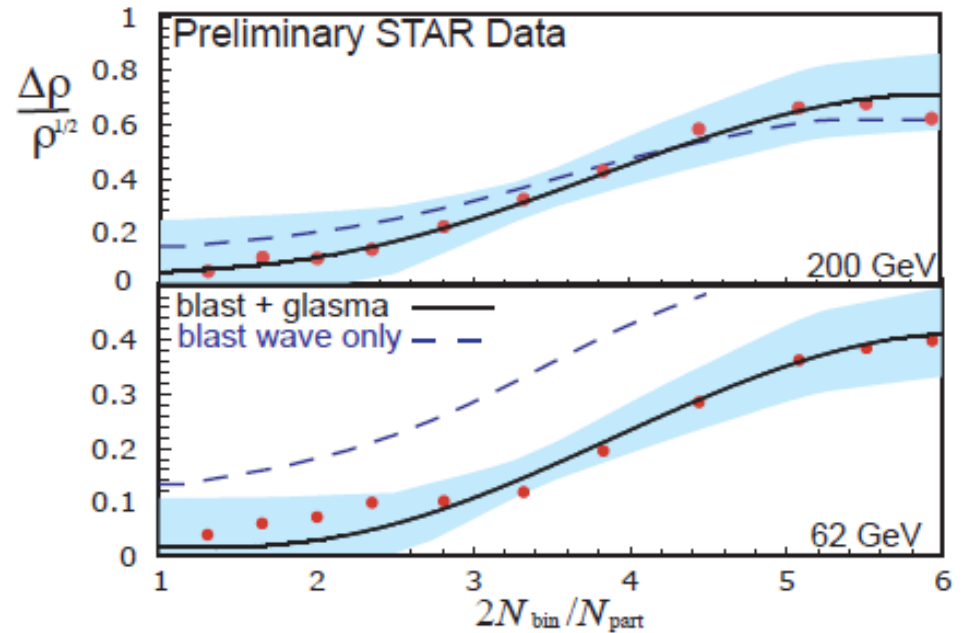
- finite width predicted
- *very* wide though, hard to see at RHIC (need  $\Delta\eta \sim 6$  !)



Dusling, Gelis, Lappi,  
Venug.: 0911.2720

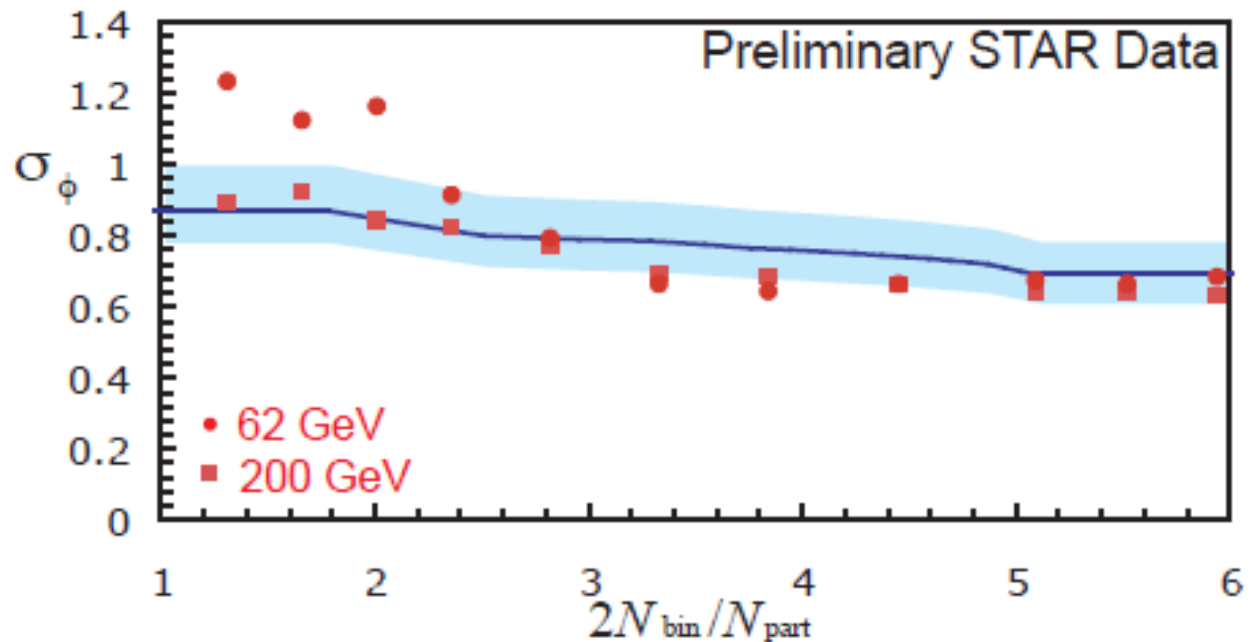
boost from radial flow

amplitude



centrality →

azimuthal  
width



Gavin, McLerran,  
Moschelli: 0806.4718